Michigan Merit Curriculum

Course/Credit Requirements

ALGEBRA I

ANCE • **RIGOR** • RELEVANCE • RELATIONSHIPS • RIGO
SHIPS • RELATIONSHIPS • RIGOR • **RELEVANCE** • R
ANCE • RIGOR • RELEVANCE • RELATIONSHIPS • RIGO
SHIPS • **RELATIONSHIPS** • RIGOR • RELEVANCE • R

1 Credit

MICHIGAN Department of Education

Michigan State Board of Education

Kathleen N. Straus, President
Bloomfield Township

John C. Austin, Vice President
Ann Arbor

Carolyn L. Curtin, Secretary
Evart

Marianne Yared McGuire, Treasurer
Detroit

Nancy Danhof, NASBE Delegate
East Lansing

Elizabeth W. Bauer
Birmingham

Reginald M. Turner
Detroit

Eileen Lappin Weiser
Ann Arbor

Governor Jennifer M. Granholm
Ex Officio

Michael P. Flanagan, Chairman
Superintendent of Public Instruction
Ex Officio

MDE Staff

Jeremy M. Hughes, Ph.D.
Deputy Superintendent/Chief Academic Officer

Dr. Yvonne Caamal Canul, Director
Office of School Improvement

Welcome

This guide was developed to assist teachers in successfully implementing the Michigan Merit Curriculum. The identified content expectations and guidelines provide a successful framework for designing curriculum, assessments and relevant learning experiences for students. Through the collaborative efforts of Governor Jennifer M. Granholm, the State Board of Education, and the State Legislature, these landmark state graduation requirements are being implemented to give Michigan students the knowledge and skills to succeed in the 21st Century and drive Michigan's economic success in the global economy. Working together, teachers can explore varied pathways to help students demonstrate proficiency in meeting the content expectations and guidelines.

Curriculum Unit Design

One of the ultimate goals of teaching is for students to acquire transferable knowledge. To accomplish this, learning needs to result in a deep understanding of content and mastery level of skills. As educational designers, teachers must use both the art and the science of teaching. In planning coherent, rigorous instructional units of study, it is best to *begin with the end in mind*.

Engaging and effective units include

- appropriate content expectations
- students setting goals and monitoring own progress
- a focus on big ideas that have great transfer value
- focus and essential questions that stimulate inquiry and connections
- identified valid and relevant skills and processes
- purposeful real-world applications
- relevant and worthy learning experiences
- varied flexible instruction for diverse learners
- research-based instructional strategies
- explicit and systematic instruction
- adequate teacher modeling and guided practice
- substantial time to review or apply new knowledge
- opportunities for revision of work based on feedback
- student evaluation of the unit
- culminating celebrations

Relevance

Instruction that is clearly relevant to today's rapidly changing world is at the forefront of unit design. Content knowledge cannot by itself lead all students to academic achievement. Classes and projects that spark student interest and provide a rationale for why the content is worth learning enable students to make connections between what they read and learn in school, their lives, and their futures. An engaging and effective curriculum provides opportunities for exploration and exposure to new ideas. Real-world learning experiences provide students with opportunities to transfer and apply knowledge in new, diverse situations.

Student Assessment

The assessment process can be a powerful tool for learning when students are actively involved in the process. Both assessment *of* learning and assessment *for* learning are essential. Reliable formative and summative assessments provide teachers with information they need to make informed instructional decisions that are more responsive to students' needs. Engagement empowers students to take ownership of their learning and builds confidence over time.

Sound assessments:

- align with learning goals
- vary in type and format
- use authentic performance tasks
- use criteria scoring tools such as rubrics or exemplars
- allow teachers and students to track growth over time
- validate the acquisition of transferable knowledge
- give insight into students' thinking processes
- cause students to use higher level thinking skills
- address guiding questions and identified skills and processes
- provide informative feedback for teachers and students
- ask students to reflect on their learning

High School Content Expectation Codes

To allow for ease in referencing expectations, each mathematics expectation has been coded by strand, standard, topic, and expectation. For example:

A1.2.3

- **A:** Algebra and Functions strand
- **A1:** Standard 1 of the Algebra and Functions strand
- **A1.2:** Topic 2 in Standard A1
- **A.1.2.3:** 3rd expectation in the 2nd topic of Standard A1

Organizational Structure

STRAND 1 **Quantitative Literacy and Logic (L)**	**STRAND 2** **Algebra and Functions (A)**
STANDARDS (and number of core expectations in each standard)	
L1: Reasoning About Numbers, Systems and Quantitative Situations (9) **L2: Calculation, Algorithms, and Estimation** (9) **L3: Measurement and Precision** (5) **L4: Mathematical Reasoning, Logic, and Proof** (10)	**A1: Expressions, Equations, and Inequalities** (16) **A2: Function** (39) **A3: Mathematical Modeling** (3)
Recommended Quantitative Literacy and Logic Expectations (3)	*Recommended Algebra and Functions Expectations (5)*

STRAND 3 **Geometry and Trigonometry (G)**	**STRAND 4** **Statistics and Probability (S)**
STANDARDS (and number of core expectations in each standard)	
G1: Figures and Their Properties (29) **G2: Relationships Between Figures** (10) **G3: Transformations of Figures in the Plane** (5)	**S1: Univariate Data—Examining Distributions** (9) **S2: Bivariate Data—Examining Relationships** (6) **S3: Samples, Surveys, and Experiments** (3) **S4: Probability Models and Probability Calculation** (4)
Recommended Geometry and Trigonometry Expectations (3)	*Recommended Statistics and Probability Expectations (6)*

Recommended Expectations

At the end of each strand, a set of recommended expectations may be listed. These extensions represent content that is desirable and valuable for all students, but attention to these items should not displace or dilute the curricular emphasis of any of the core expectations. Teachers are encouraged to incorporate the recommended expectations into their instruction when their students have a solid foundation and are ready for enrichment or advanced learning. Recommended expectations will not be tested on the Michigan Merit Exam or on future high school subject credit assessments.

Coding Note: Recommended expectations have an * preceding their code.

Organization of this Document

In the mathematics credit requirement documents, the expectations are organized by strand and standard underneath topic headings. The organization in no way implies an instructional sequence. Curriculum personnel or teachers are encouraged to organize these topics and expectations in a manner that encourages connections between strands and among topics with a strand.

Introduction to Algebra I

Algebra is not only a theoretical tool for analyzing and describing mathematical relationships, but it is also a powerful tool for the mathematical modeling and solving of real-world problems. These problems can be found all around us: the workplace, the sciences, technology, engineering, and mathematics.

Algebra I Goal Statement

Algebra I builds upon a number of key algebraic topics assumed to have been developed in the middle grades, namely a deep knowledge of linear patterns of change and familiarity with nonlinear patterns such as exponential and quadratic. It is expected that students entering Algebra I are able to recognize and solve mathematical and real-world problems involving linear relationships and to make sense of and move fluently among the graphic, numeric, symbolic, and verbal representations of these patterns. In addition, students should be able to apply this knowledge to quadratic and other simple functions.

Algebra I builds on the increasingly generalized approach to the study of functions and representations begun in the middle grades. This is done by broadening the study of linear relationships to include piecewise functions such as absolute value and greatest integer, systems of equations with three unknowns, formalized function notation and recursive representations, and the development of bivariate data analysis topics such as linear regression and correlation. In addition, their knowledge of exponential and quadratic function families is extended and deepened with the inclusion of topics such as rules of exponentiation (including rational exponents), introduction to logarithmic patterns as the inverse of exponential equations, and use of standard and vertex forms for quadratic equations. Students will also develop their knowledge of power (including roots, cubics, and quartics) and polynomial patterns of change and the applications they model.

In addition to deepening and extending the student's knowledge of the algebra strand, Algebra I also draws upon and connects to topics related to number and geometry by including the formalized study of the real number system and its properties, and by introducing elementary number theory. Finally, Algebra I expands the number domain from real to complex numbers, opening up the opportunity to study operations over the set of complex numbers, vector representations, and applications.

Throughout Algebra I and II, students will experience mathematics generally, and algebra in particular, not only as the theoretical study of mathematical patterns and relationships but also as a language that allows us to make sense of mathematical symbols. Finally, students will develop an understanding that algebraic thinking is an accessible and powerful tool that can be used to model and solve real-world problems.

Algebra I Content Expectations Outline

STANDARD L1: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE SITUATIONS

- L1.1 Number Systems and Number Sense
- L1.2 Representations and Relationships

STANDARD L2: CALCULATION, ALGORITHMS, AND ESTIMATION

- L2.1 Calculation Using Real and Complex Numbers

STANDARD L3: MEASUREMENT AND PRECISION

- L3.1 Measurement Units, Calculations, and Scales

STANDARD A1: EXPRESSIONS, EQUATIONS, AND INEQUALITIES

- A1.1 Construction, Interpretation, and Manipulation of Expressions (linear, quadratic, polynomial, rational, power, exponential, and logarithmic)
- A1.2 Solutions of Equations and Inequalities (linear, exponential, logarithmic, quadratic, power, polynomial, and rational)

STANDARD A2: FUNCTIONS

- A2.1 Definitions, Representations, and Attributes of Functions
- A2.2 Operations and Transformations
- A2.3 Families of Functions (linear, quadratic, polynomial, power, exponential, and logarithmic)
- A2.4 Lines and Linear Functions
- A2.5 Exponential and Logarithmic Functions
- A2.6 Quadratic Functions
- A2.7 Power Functions (including roots, cubics, quartics, etc.)
- A2.8 Polynomial Functions

STANDARD A3: MATHEMATICAL MODELING

- A3.1 Models of Real-world Situations Using Families of Functions (linear, quadratic, exponential, and power)

STANDARD S2: BIVARIATE DATA-EXAMINING RELATIONSHIPS

- S2.1 Scatterplots and Correlation
- S2.2 Linear Regression

CONTENT EXPECTATIONS FOR ALGEBRA I

STANDARD L1: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE LITERACY

L1.1 Number Systems and Number Sense

L1.1.1 Know the different properties that hold in different number systems and recognize that the applicable properties change in the transition from the positive integers to all integers, to the rational numbers, and to the real numbers.

L1.1.2 Explain why the multiplicative inverse of a number has the same sign as the number, while the additive inverse of a number has the opposite sign.

L1.1.3 Explain how the properties of associativity, commutativity, and distributivity, as well as identity and inverse elements, are used in arithmetic and algebraic calculations.

L1.1.4 Describe the reasons for the different effects of multiplication by, or exponentiation of, a positive number by a number less than 0, a number between 0 and 1, and a number greater than 1.

L1.1.5 Justify numerical relationships (e.g., show that the sum of even integers is even; that every integer can be written as $3m+k$, where k is 0, 1, or 2, and m is an integer; or that the sum of the first n positive integers is $n\ (n+1)/2$).

L1.2 Representations and Relationships

L1.2.2 Interpret representations that reflect absolute value relationships (e.g. $|x - a| \leq b$, *or* $a \pm b$) in such contexts as error tolerance.

L1.2.4 Organize and summarize a data set in a table, plot, chart, or spreadsheet; find patterns in a display of data; understand and critique data displays in the media.

STANDARD L2: CALCULATION, ALGORITHMS, AND ESTIMATION

L2.1 Calculation Using Real and Complex Numbers

L2.1.1 Explain the meaning and uses of weighted averages (e.g., GNP, consumer price index, grade point average).

L2.1.2 Calculate fluently with numerical expressions involving exponents. Use the rules of exponents, and evaluate numerical expressions involving rational and negative exponents, and transition easily between roots and exponents.

L2.1.3 Explain the exponential relationship between a number and its base 10 logarithm and use it to relate rules of logarithms to those of exponents in expressions involving numbers.

L2.1.4 Know that the complex number i is one of two solutions to $x^2 = -1$.

L2.1.5 Add, subtract, and multiply complex numbers. Use conjugates to simplify quotients of complex numbers.

L2.1.6 Recognize when exact answers aren't always possible or practical. Use appropriate algorithms to approximate solutions to equations (e.g., to approximate square roots).

STANDARD L3: MEASUREMENT AND PRECISION

L3.1 Measurement Units, Calculations, and Scales

L3.1.2 Describe and interpret logarithmic relationships in such contexts as the Richter scale, the pH scale, or decibel measurements (e.g., explain why a small change in the scale can represent a large change in intensity). Solve applied problems.

STANDARD A1: EXPRESSIONS, EQUATIONS, AND INEQUALITIES

A1.1 Construction, Interpretation, and Manipulation of Expressions (linear, quadratic, polynomial, rational, power, exponential, and logarithmic)

A1.1.1 Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.

A1.1.2 Know the definitions and properties of exponents and roots and apply them in algebraic expressions.

A1.1.3 Factor algebraic expressions using, for example, greatest common factor, grouping, and the special product identities (e.g., differences of squares and cubes).

A1.1.6 Use the properties of exponents and logarithms, including the inverse relationship between exponents and logarithms, to transform exponential and logarithmic expressions into equivalent forms.

A1.2 Solutions of Equations and Inequalities (linear, exponential, logarithmic, quadratic, power, polynomial, and rational)

A1.2.1 Write and solve equations and inequalities with one or two variables to represent mathematical or applied situations.

A1.2.2 Associate a given equation with a function whose zeros are the solutions of the equation.

A1.2.3 Solve linear and quadratic equations and inequalities, including systems of up to three linear equations with three unknowns. Justify steps in the solutions, and apply the quadratic formula appropriately.

A1.2.4 Solve absolute value equations and inequalities (e.g., solve $|x - 3| \leq 6$) and justify.

A1.2.6 Solve power equations (e.g., $(x + 1)^3 = 8$) and equations including radical expressions (e.g., $\sqrt{3x - 7} = 7$), justify steps in the solution, and explain how extraneous solutions may arise.

A1.2.8 Solve an equation involving several variables (with numerical or letter coefficients) for a designated variable. Justify steps in the solution.

STANDARD A2: FUNCTIONS

A2.1 Definitions, Representations, and Attributes of Functions

A2.1.1 Recognize whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function and identify its domain and range.

A2.1.2 Read, interpret, and use function notation and evaluate a function at a value in its domain.

A2.1.3 Represent functions in symbols, graphs, tables, diagrams, or words and translate among representations.

A2.1.4 Recognize that functions may be defined by different expressions over different intervals of their domains. Such functions are piecewise-defined (e.g., absolute value and greatest integer functions).

A2.1.5 Recognize that functions may be defined recursively. Compute values of and graph simple recursively defined functions (e.g., $f(0) = 5$, and $f(n) = f(n\text{-}1) + 2$).

A2.1.6 Identify the zeros of a function and the intervals where the values of a function are positive or negative. Describe the behavior of a function as x approaches positive or negative infinity, given the symbolic and graphical representations.

A2.1.7 Identify and interpret the key features of a function from its graph or its formula(e), (e.g., slope, intercept(s), asymptote(s), maximum and minimum value(s), symmetry, and average rate of change over an interval).

A2.2 Operations and Transformations

A2.2.1 Combine functions by addition, subtraction, multiplication, and division.

A2.2.2 Apply given transformations (e.g., vertical or horizontal shifts, stretching or shrinking, or reflections about the x- and y-axes) to basic functions and represent symbolically.

A2.2.3 Recognize whether a function (given in tabular or graphical form) has an inverse and recognize simple inverse pairs (e.g., $f(x) = x^3$ and $g(x) = x^{1/3}$).

A2.3 Families of Functions (linear, quadratic, polynomial, power, exponential, and logarithmic)

A2.3.1 Identify a function as a member of a family of functions based on its symbolic or graphical representation. Recognize that different families of functions have different asymptotic behavior at infinity and describe these behaviors.

A2.3.2 Describe the tabular pattern associated with functions having constant rate of change (linear) or variable rates of change.

A2.4 Lines and Linear Functions

A2.4.1 Write the symbolic forms of linear functions (standard [i.e., $Ax + By = C$, where $B \neq 0$], point-slope, and slope-intercept) given appropriate information and convert between forms.

A2.4.2 Graph lines (including those of the form $x = h$ and $y = k$) given appropriate information.

A2.4.3 Relate the coefficients in a linear function to the slope and x- and y-intercepts of its graph.

A2.4.4 Find an equation of the line parallel or perpendicular to given line through a given point. Understand and use the facts that nonvertical parallel lines have equal slopes and that nonvertical perpendicular lines have slopes that multiply to give -1.

A2.5 Exponential and Logarithmic Functions

A2.5.1 Write the symbolic form and sketch the graph of an exponential function given appropriate information (e.g., given an initial value of 4 and a rate of growth of 1.5, write $f(x) = 4\ (1.5)^x$).

A2.5.4 Understand and use the fact that the base of an exponential function determines whether the function increases or decreases and how base affects the rate of growth or decay.

A2.5.5 Relate exponential and logarithmic functions to real phenomena, including half-life and doubling time.

A2.6 Quadratic Functions

A2.6.1 Write the symbolic form and sketch the graph of a quadratic function given appropriate information (e.g., vertex, intercepts, etc.).

A2.6.2 Identify the elements of a parabola (vertex, axis of symmetry, and direction of opening) given its symbolic form or its graph and relate these elements to the coefficient(s) of the symbolic form of the function.

A2.6.3 Convert quadratic functions from standard to vertex form by completing the square.

A2.6.4 Relate the number of real solutions of a quadratic equation to the graph of the associated quadratic function.

A2.6.5 Express quadratic functions in vertex form to identify their maxima or minima and in factored form to identify their zeros.

A2.7 Power Functions (including roots, cubics, quartics, etc.)

A2.7.1 Write the symbolic form and sketch the graph of power functions.

A2.7.2 Express direct and inverse relationships as functions (e.g., $y = kx^n$ and $y = kx^{-n}$, $n > 0$) and recognize their characteristics (e.g., in $y = x^3$, note that doubling x results in multiplying y by a factor of 8).

A2.7.3 Analyze the graphs of power functions, noting reflectional or rotational symmetry.

A2.8 Polynomial Functions

A2.8.1 Write the symbolic form and sketch the graph of simple polynomial functions.

A2.8.2 Understand the effects of degree, leading coefficient, and number of real zeros on the graphs of polynomial functions of degree greater than 2.

A2.8.3 Determine the maximum possible number of zeroes of a polynomial function and understand the relationship between the x-intercepts of the graph and the factored form of the function.

STANDARD A3: MATHEMATICAL MODELING

A3.1 Models of Real-world Situations Using Families of Functions (linear, quadratic, exponential and power)

Example: An initial population of 300 people grows at 2% per year. What will the population be in 10 years?

A3.1.1 Identify the family of function best suited for modeling a given real-world situation [e.g., quadratic functions for motion of an object under the force of gravity or exponential functions for compound interest. *In the example above, recognize that the appropriate general function is exponential* ($P = P_0 a^t$)].

A3.1.2 Adapt the general symbolic form of a function to one that fits the specifications of a given situation by using the information to replace arbitrary constants with numbers. *In the example above, substitute the given values* $P_0 = 300$ *and* $a = 1.02$ *to obtain* $P = 300(1.02)^t$.

A3.1.3 Using the adapted general symbolic form, draw reasonable conclusions about the situation being modeled. *In the example above, the exact solution is 365.698, but for this problem, an appropriate approximation is 365.*

RECOMMENDED:

***A3.1.4** Use methods of linear programming to represent and solve simple real-life problems.

STANDARD S2: BIVARIATE DATA-EXAMINING RELATIONSHIPS

S2.1 Scatterplots and Correlation

S2.1.1 Construct a scatterplot for a bivariate data set with appropriate labels and scales.

S2.1.2 Given a scatterplot, identify patterns, clusters, and outliers. Recognize no correlation, weak correlation, and strong correlation.

S2.1.3 Estimate and interpret Pearson's correlation coefficient for a scatterplot of a bivariate data set. Recognize that correlation measures the strength of linear association.

S2.1.4 Differentiate between correlation and causation. Know that a strong correlation does not imply a cause-and-effect relationship. Recognize the role of lurking variables in correlation.

S2.2 Linear Regression

S2.2.1 For bivariate data that appear to form a linear pattern, find the least squares regression line by estimating visually and by calculating the equation of the regression line. Interpret the slope of the equation for a regression line.

S2.2.2 Use the equation of the least squares regression line to make appropriate predictions.

NOTES

NOTES

Michigan Department of Education
Office of School Improvement
Dr. Yvonne Caamal Canul, Director
(517) 241-3147 www.michigan.gov/mde